

# Synthesis Techniques for High Performance Octave Bandwidth $180^\circ$ Analog Phase Shifters

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**Abstract**—Novel techniques for synthesizing  $180^\circ$  analog reflection-type phase shifters, with ultra-low phase and amplitude error characteristics, over a very wide bandwidth, are presented. The novel approach of cascading stages, where the nonlinear performance of each stage compliments those of the others, results in a significant advance in the linearity performance of traditional reflection-type phase shifters. In this paper, it is shown by theoretical analysis that three conditions must be satisfied by the reflection terminations, in order to achieve the desired response. The theoretical conditions and subsequent design equations are given. Simulation results for a 2-stage *Ku*-band cascaded-match reflection-type phase shifter show that a very low maximum phase error and amplitude error of  $\pm 2.4^\circ$  and  $\pm 0.21$  dB, respectively, can be achieved over a full octave bandwidth. Since the complexity of the overall topology is reduced to a minimum, the device appears insensitive to process variations and ideal for both hybrid and MMIC technologies.

## I. INTRODUCTION

THE reflection-type phase shifter (RTPS) was first introduced more than three decades ago by Hardin [1]. With reference to Fig. 1, a directional coupler produces incident voltage waves at the coupled and direct ports. These waves are reflected by identical impedances,  $Z_T$ , terminating the ports. The resulting reflected waves then combine at the input and isolation ports. If lossless reflection terminations are connected to an ideal 3 dB quadrature directional coupler, the resulting voltage wave vectors at the input port cancel each other out, while those at the isolation port re-enforce each other. In other words, all the power entering the input port will emerge from the isolation port. If the reflection terminations have a voltage dependent reactance, in the form of varactor-type devices or active inductors, the relative phase difference between the output voltage wave vector and the input voltage wave vector can be electronically varied, therefore, creating a relative shift in the phase angle of the overall voltage transmission coefficient.

Hardin proposed a simple series-tuned circuit for the reflection terminations, incorporating a single varactor diode in each. The resulting frequency response of the relative phase shift has a single hump. This hump can produce a maximum bandwidth of approximately 5–10%, for a phase error of  $\pm 5^\circ$  between a  $0^\circ$  and  $180^\circ$  relative phase

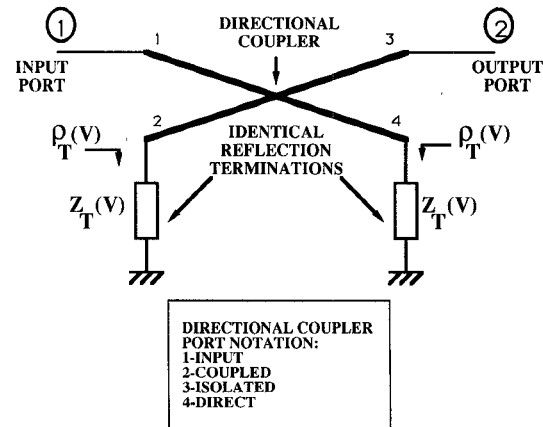


Fig. 1. Ideal 1-stage reflection-type phase shifter.

shift. This idea was further developed by Searing [2] and Garver [3] without much improvement to the bandwidth. Attempts to widen the bandwidth were proposed by Henoch *et al.* [4] and Ulriksson [5]. Here a parallel resonance mode was synthesized in the reflection termination—by connecting two series-tuned circuits, with different resonant frequencies, in parallel. A transmission line was then inserted between the reflection termination and the circulator directional coupler, in order to produce a double humped frequency response. This technique can increase the bandwidth by a factor of two. Also, the component values of the reflection terminations could only be determined by intensive optimization and human judgement. In recent years, a number of variations to the basic reflection terminations have been reported without significant advances in the phase shifter's overall performance [6]–[12].

A novel method for increasing the bandwidth is to combine the simplicity of Hardin's approach with the type of rippled frequency response produced by the latter approaches. Here, the phase shifter is split-up into matched cascaded stages. In order to achieve a full octave bandwidth, the optimum number of stages required by this cascaded-match reflection-type phase shifter (CMRTPS) is two; while a three stage design may be required to achieve a decade bandwidth. An ideal 2-stage CMRTPS is illustrated in Fig. 2.

For a 2-stage design each stage is similar to the one proposed by Hardin, except that the resonant frequency of the series-tuned reflection termination of the second

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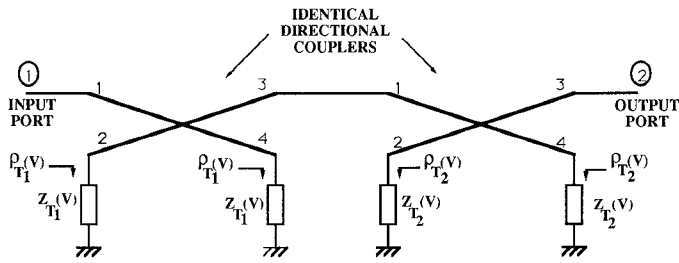


Fig. 2. Ideal 2-stage cascaded-match reflection-type phase shifter.

stage,  $\omega_{OS_2}$  is always much greater than that of the first stage,  $\omega_{OS_1}$ .

For a voltage-controlled phase shifter, the relative phase shift,  $\Delta \angle S_{21}(\omega, V)|_{PS}$ , can be defined as follows:

$$\Delta \angle S_{21}(\omega, V)|_{PS} = \angle \left( \frac{S_{21}(\omega, V)|_{PS}}{S_{21}(\omega, 0)|_{PS}} \right)$$

where

$V$  = applied bias potential and  $V_1 \leq V \leq V_2$

$V_1$  = minimum applied bias potential ( $V_1 = 0$  in most cases)

$V_2$  = maximum applied bias potential

$S_{21}(\omega, V)|_{PS}$  = voltage transmission coefficient of the phase shifter at  $V$  bias

$S_{21}(\omega, 0)|_{PS}$  = voltage transmission coefficient of the phase shifter at zero bias.

With ideal 3 dB quadrature coupling, maintained across the octave bandwidth, the relative phase shift would simply be the superposition of the relative phase shifts in the reflection coefficients,  $\rho_{T_1}$  and  $\rho_{T_2}$ , produced by the reflection terminations of both stages, i.e.:

$$\Delta \angle S_{21}(\omega, V)|_{PS} = \Delta \angle \rho_{T_1}(\omega, V) + \Delta \angle \rho_{T_2}(\omega, V)$$

where

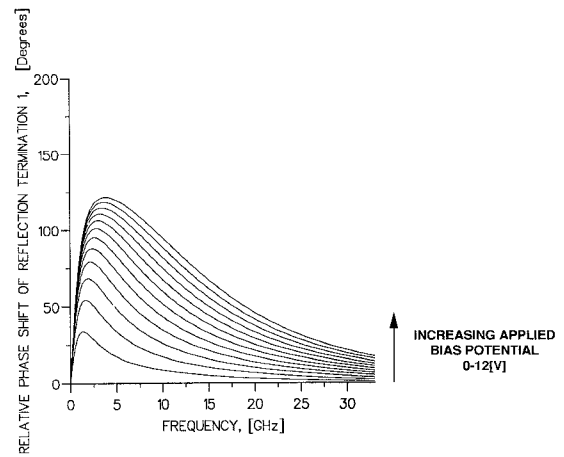
$$\Delta \angle \rho_{T_1}(\omega, V) = \angle \left( \frac{\rho_{T_1}(\omega, V)}{\rho_{T_1}(\omega, 0)} \right)$$

and

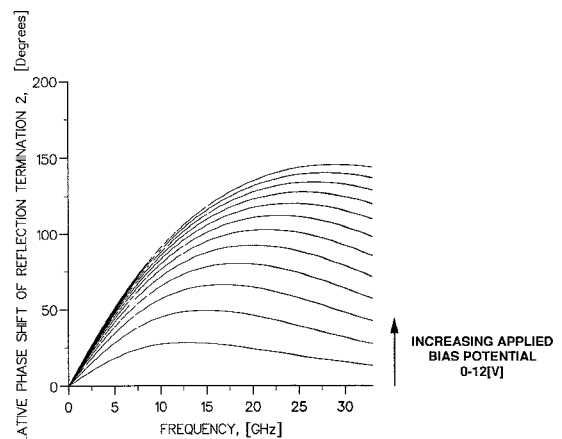
$$\Delta \angle \rho_{T_2}(\omega, V) = \angle \left( \frac{\rho_{T_2}(\omega, V)}{\rho_{T_2}(\omega, 0)} \right)$$

where  $\rho_{T_1(2)}(\omega, V)$  = voltage reflection coefficient of  $Z_{T_1(2)}$  at  $V$  bias and  $\rho_{T_1(2)}(\omega, 0)$  = voltage reflection coefficient of  $Z_{T_1(2)}$  at zero bias.

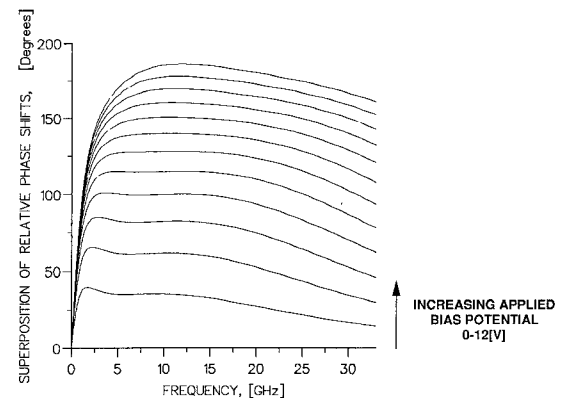
If the hump produced by the first stage is centered below the lowest frequency of the desired frequency range and that produced by the second stage near the highest frequency, then with the appropriate component values the negative gradients of the first stage will compensate for the positive gradients of the second stage, within the desired frequency range. The resulting frequency response will be flat over the desired frequency range, for relative phase shifts up to and exceeding  $180^\circ$ .



(a)



(b)



(c)

Fig. 3. Relative phase shift for the: (a) reflection coefficient of  $Z_{T_1}$ ; (b) reflection coefficient of  $Z_{T_2}$ ; (c) phase shifter with ideal 3 dB quadrature coupling maintained at all frequencies.

Fig. 3 shows the relative phase shift in  $\rho_{T_1}$  and  $\rho_{T_2}$ , produced by  $Z_{T_1}$  and  $Z_{T_2}$ , respectively, and the subsequent superposition of these frequency responses for a  $Ku$ -band CMRTPS, using ideal 3 dB quadrature couplers. Within an octave bandwidth—having a lower band-edge frequency  $F_1 = 10$  GHz, an upper band-edge frequency  $F_2 = 20$  GHz and a mid-band frequency  $F_O = 15$  GHz—the superposition of the frequency responses have negative

gradients. It will be shown in a subsequent design example that the near linear decrease in the isolation, with respect to frequency, of the 4-finger Lange couplers can increase the gradient to its desired zero value.

## II. SIMULATION

The directional coupler can be assumed to be symmetrical, reciprocal and lossless—which is a good approximation in most cases.

Therefore, with the appropriate expressions obtained for a Lange couplers' even mode effective permittivity,  $\epsilon_{ee}(f)$ ; odd mode effective permittivity,  $\epsilon_{eo}(f)$ ; even mode capacitance,  $C_{oe}(f)$ ; odd mode capacitance,  $C_{oo}(f)$ , and the finger length,  $l$ , [15] the 4-port steady-state  $S$ -parameters  $S_{ij}$  (where  $i, j \in [1, 4]$ ), can be accurately determined [16].

In some circumstances, it may not be possible to cascade the 2-stages of the CMRTPS without some form of matched, lossless, transmission line—due to restricted layout considerations. If this stage isolation line (SIL) is to be inserted between the stages, its model must be included in the simulations.

$S$ -parameters resulting from transient, time-domain, signals can clearly describe the direction of transient power flow around the topology. Using the 4-port steady-state  $S$ -parameters of the directional coupler, the transient 2-port  $S$ -parameters for the 2-stage CMRTPS,  $S_{mn}|_{PS}$  (where  $m, n \in [1, 2]$ ), can be expressed as follows:

$$\begin{aligned} S_{11}|_{PS} = S_{22}|_{PS} &= S_{11A} + (S_{21A}S_{21C})^2 \\ &\cdot \sum_{i=0}^p S_{11B}^{(i+1)} S_{22A}^i \\ S_{21}|_{PS} = S_{12}|_{PS} &= S_{21A}S_{21C}S_{21B} \\ &\cdot \sum_{i=0}^p (S_{11B}S_{22A})^i \end{aligned}$$

where

$$\begin{aligned} S_{mnA} &= \text{Steady-state } S\text{-parameter for the first stage} \\ S_{mnB} &= \text{Steady-state } S\text{-parameter for the second stage} \\ S_{mnC} &= \text{Steady-state } S\text{-parameter for the SIL} \\ p &= \text{order of inter-stage reflection.} \end{aligned}$$

Here,

$$\begin{aligned} S_{11A(B)} = S_{22A(B)} &= S_{11} + (S_{21}^2 + S_{41}^2) \\ &\cdot \sum_{i=0}^q \rho_{T(2)}^{(i+1)} S_{11}^i \\ S_{21A(B)} = S_{12A(B)} &= S_{31} + 2S_{21}S_{41} \\ &\cdot \sum_{i=0}^q \rho_{T(2)}^{(i+1)} S_{11}^i \end{aligned}$$

where  $q$  = order of intra-stage reflection

$$S_{21C} = e^{-j\theta_c(\omega)}$$

where  $\theta_c(\omega)$  = electrical length of the SIL where

$$\theta_c(\omega) = \frac{\omega}{\omega_0} \cdot \theta(\omega_0)$$

and

$$\theta(\omega_0) = \frac{\omega_0 \sqrt{\epsilon_e(\omega_0)}}{c} \cdot l_c$$

where  $\epsilon_e$  = effective permittivity of the SIL and,  $l_c$  = physical length of the SIL.

For the true steady-state 2-port  $S$ -parameters, both  $p$  and  $q$  must tend to infinity, resulting in the following closed-form expressions:

$$\begin{aligned} S_{11}|_{PS} = S_{22}|_{PS} &= S_{11A} + \frac{(S_{21A}S_{21C})^2 S_{11B}}{1 - S_{22A}S_{11B}} \\ S_{21}|_{PS} = S_{12}|_{PS} &= \frac{S_{21A}S_{21C}S_{21B}}{1 - S_{22A}S_{11B}} \end{aligned}$$

where

$$S_{11A(B)} = S_{22A(B)} = S_{11} + \frac{\rho_{T(2)}(S_{21}^2 + S_{41}^2)}{1 - \rho_{T(2)}S_{11}}$$

and

$$S_{21A(B)} = S_{12A(B)} = S_{31} + \frac{2\rho_{T(2)}S_{21}S_{41}}{1 - \rho_{T(2)}S_{11}}$$

## III. SYNTHESIS

In order to synthesize the 2-stage octave bandwidth CMRTPS, the reflection terminations and directional couplers can be treated separately. However, it should be noted that the empirical expressions obtained for the component values within the reflection terminations are a function of the characteristics of the directional couplers and the presence of the other stage.

In theory, it will be seen that the exact frequency characteristics obtained with the CMRTPS covering an octave bandwidth can be scaled to cover any chosen octave bandwidth. This is achieved by the appropriate scaling of the component values within the reflection terminations and the physical length of the directional coupler.

### Directional Couplers

Two-stage octave bandwidth CMRTPS's require octave bandwidth directional couplers—with tight coupling within the bandwidth.

In order to achieve a return loss within acceptable levels, 3 dB coupling is required at the frequency of maximum coupling,  $F_C$ , —with a 3 dB coupling imbalance,  $C_I \leq \pm 1.5$  dB, within the octave bandwidth. Many types of directional coupler qualify, such as the common 4-finger Lange coupler. However, the narrow band 1-section branch line coupler has also been used in wideband RTPS's [7], [8].

### Reflection Termination

A single flat level relative phase shift frequency response is quite easy to achieve over an octave bandwidth. As the number of required flat levels increase, the difficulties in achieving these flat levels also increases. In order to obtain a low phase and amplitude error over the full octave bandwidth, at all levels from  $0^\circ$  to over  $180^\circ$ , the following conditions have been found and must be satisfied by the reflection terminations:

a) The resonant frequency of the second stage reflection termination must correctly scale that of the first stage, at the zero bias potential.

b) The change in the resonant frequency of the second stage reflection termination must correctly track the change in that of the first stage.

c) The reflection coefficient of the second stage reflection termination must have a phase angle whose gradient, with respect to frequency, correctly scales that of the first stage, at their respective frequency of resonance.

These conditions will now be examined in detail.

#### A. Initial Resonance Scaling

At zero bias, linear relationships exist between the resonant frequency of the first and second stage reflection terminations and the mid-band frequency, i.e.:

$$\frac{\omega_{OS_1}(0)}{\omega_0} \equiv A_1 \quad \text{and} \quad \frac{\omega_{OS_2}(0)}{\omega_0} \equiv A_2$$

where

$$\begin{aligned} \omega_{OS_1}(0) &= \text{series resonant frequency of } Z_{T_1} \text{ at zero bias} \\ \omega_{OS_2}(0) &= \text{series resonant frequency of } Z_{T_2} \text{ at zero bias} \\ \omega_0 &= 2\pi F_0 \\ A_1 &= \text{frequency scaling factor for } Z_{T_1} \\ A_2 &= \text{frequency scaling factor for } Z_{T_2}. \end{aligned}$$

Therefore,

$$\frac{\omega_{OS_2}(0)}{\omega_{OS_1}(0)} \equiv A_3$$

i.e., initial resonance scaling.

The following design equations can now be obtained:

$$C_{1T}(0) = \frac{1}{(A_1 \omega_0)^2 L_1} \quad \text{and} \quad C_{2T}(0) = \frac{1}{(A_2 \omega_0)^2 L_2}$$

where  $C_{1T}(0)$  = total series capacitance in  $Z_{T_1}$  at zero bias and  $C_{2T}(0)$  = total series capacitance in  $Z_{T_2}$  at zero bias.

#### B. Resonance Tracking

The tuning ratio,  $m(V)$ , of a series resonant tuned circuit is defined as

$$m(V) = \frac{\omega_{OS}(V)}{\omega_{OS}(0)}$$

Chip varactors have negligible parasitic reactive components below 10 GHz. If chip varactors are employed to

vary the resonant frequency, with voltage-invariant series inductors, it can be shown that:

$$m(V) = \sqrt{\frac{C(0)}{C(V)}}$$

where  $C(V)$  = junction capacitance of the chip varactor with  $V$  bias and  $C(0)$  = junction capacitance of the chip varactor with zero bias.

In general, the junction capacitance can be expressed as

$$C(V) = \frac{C(0)}{(1 + V/\phi)^\gamma}$$

where

$$\begin{aligned} \phi &= \text{built-in barrier potential} \\ \gamma &= \text{slope exponent.} \end{aligned}$$

Therefore,

$$m(V) = \sqrt{(1 + V/\phi)^\gamma}$$

As  $m(V)$  increases, the size of the hump in the relative phase shift frequency response, for a single stage reflection-type phase shifter, also increases. The rate at which this size increases, with respect to an increase in the bias potential, increases as  $\omega_{OS}(0)$  increases. Since  $\omega_{OS_2}(0) > \omega_{OS_1}(0)$ , for a 2-stage CMRTPS, if an identical bias is applied to all varactors the reflection termination of the second stage must incorporate a specific amount of decoupling, in order to reduce the rate in which its peak increases.

The decoupling can be realized by a voltage invariant series coupling capacitor,  $C_C$ . This capacitor has the second function of dc blocking—if placed between the reflection termination and the corresponding port of its directional coupler. Dc blocking may be required to isolate the applied bias from the other stage and the input and output ports of the phase shifter.

With the introduction of  $C_C$ , the total series capacitance of a reflection termination,  $C_T(V)$ , is now given by

$$C_T(V) = \frac{C_C C(V)}{C_C + C(V)}$$

The amount of coupling between the reflection termination and its directional coupler can be expressed by the reflection termination coupling coefficient,  $k$ , defined as

$$k = \frac{C_C}{C_C + C(0)}$$

where  $k = 0$  for an uncoupled reflection termination and  $k = 1$  for a fully coupled reflection termination.

In most cases, the reflection termination of the first stage can be fully coupled. This has the advantage of making the value of  $C_C$ , non-critical. As a good rule-of-thumb, make  $C_C \geq 20 \cdot C_1(V)$ , therefore,  $k_1 \approx 1$ .

If identical technologies and slope exponents are used in the junction capacitances of both stages, i.e.,  $\phi_1 = \phi_2$

=  $\phi$  and  $\gamma_1 = \gamma_2 = \gamma$ , respectively, it can be shown that when the first stage reflection termination is fully coupled the tuning ratio of  $Z_{T_2}$ ,  $m_2(V)$ , is given by

$$m_2(v) = \sqrt{k_2 m_1^2(V) - k_2 + 1}$$

i.e., resonance tracking

where  $m_1(V)$  = tuning ratio of  $Z_{T_1}$  and

$$m_1(V) = \sqrt{(1 + V/\phi)^\gamma}.$$

Therefore,

$$\frac{\partial m_2(V)}{\partial V} = \frac{\gamma k_2 (1 + V/\phi)^{(\gamma-1)}}{2\phi \sqrt{k_2 (1 + V/\phi)^\gamma - k_2 + 1}}$$

and

$$\frac{\partial m_1(V)}{\partial V} = \frac{\gamma}{2\phi} (1 + V/\phi)^{(\gamma/2-1)}.$$

Therefore,

$$0 \leq \frac{\partial m_2(V)}{\partial V} \leq \frac{\partial m_1(V)}{\partial V}.$$

The lower limit is reached when  $k_2 = 0$  and the upper when  $k_2 = 1$ .

In order to achieve a low phase error at all levels, over an octave bandwidth, it is found that  $k$  should be just under 0.5, resulting in  $\partial m_2(V)/\partial V \approx \frac{1}{2} \cdot \partial m_1(V)/\partial V$ .

### C. Phase Gradient Scaling

The reflection coefficient of a reflection termination,  $\rho_T$ , is defined as

$$\rho_T = \frac{Z_T - Z_O}{Z_T + Z_O}$$

where  $Z_T$  = reflection termination impedance and  $Z_O$  = reference impedance standard for the system ( $Z_O = 50 \Omega$ , in most cases).

For a series R-L-C reflection termination, the following is obtained:

$$\rho_T = \frac{(R_S^2 - Z_O^2 + X_S^2) + j(2Z_O X_S)}{(R_S + Z_O)^2 + X_S^2}$$

where

$$\begin{aligned} X_S &= X_L + X_C \\ R_S &= \text{total series loss resistance} \\ X_L &= \text{reactance of the total series inductance} \\ X_C &= \text{reactance of the total series capacitance.} \end{aligned}$$

Therefore,

$$|\rho_T| = \sqrt{\left(\frac{(R_S^2 - Z_O^2 + X_S^2)}{(R_S + Z_O)^2 + X_S^2}\right)^2 + \left(\frac{2Z_O X_S}{(R_S + Z_O)^2 + X_S^2}\right)^2}$$

and

$$\angle \rho_T = \tan^{-1} \left( \frac{2Z_O X_S}{R_S^2 - Z_O^2 + X_S^2} \right).$$

Therefore,

$$\frac{\partial \angle \rho_T}{\partial \omega} = -\frac{2Z_O(X_L - X_C)}{\omega} \cdot \left( \frac{2X_S^2 - (R_S^2 - Z_O^2 + X_S^2)}{(2Z_O X_S)^2 + (R_S^2 - Z_O^2 + X_S^2)^2} \right).$$

At resonance:

$$\left. \frac{\partial \angle \rho_T}{\partial \omega} \right|_{\omega_{OS}} = -\frac{4Z_O L}{Z_O^2 - R_S^2} \approx -\frac{4L}{Z_O},$$

with high-Q chip varactors.

Therefore,

$$\left. \frac{\partial \angle \rho_T}{\partial \omega} \right|_{\omega_{OS}}$$

$$\equiv K \cdot L, \quad \text{i.e., a constant, irrespective of } \omega_{OS}$$

where  $K$  = phase gradient constant.

For the 2-stage topology, it is found that a linear relationship must exist between the angle of reflection coefficient gradients, at their respective resonant frequencies, i.e.:

$$\frac{\partial \angle \rho_{T_1} / \partial \omega |_{\omega_{OS_1}}}{\partial \angle \rho_{T_2} / \partial \omega |_{\omega_{OS_2}}}$$

$$\equiv A_4, \quad \text{i.e., phase gradient scaling.}$$

Therefore, the following relationship can be obtained:

$$L_1 = \left( A_4 \frac{K_2}{K_1} \right) L_2 \approx A_4 L_2,$$

with high-Q chip varactors and identical directional couplers where  $K_1$  = phase gradient constant for  $Z_{T_1}$  and  $K_2$  = phase gradient constant for  $Z_{T_2}$ .

In order to achieve a low phase error, at all levels over an octave, or greater, bandwidth, it is found that  $|\partial \angle \rho_{T_2} / \partial \omega |_{\omega_{OS_2}}$  must be less than  $|\partial \angle \rho_{T_1} / \partial \omega |_{\omega_{OS_1}}$ , i.e.,  $A_4 > 1$ .

As  $A_4$  is increased beyond unity, the potential bandwidth continues to increase and the rate in which the peak increases effectively decreases in the second stage of the CMRTPS. The latter is similar to reflection termination decoupling.

It has also been found that a linear relationship exists between  $\partial \angle \rho_{T_1} / \partial \omega |_{\omega_{OS_1}}$  and  $\omega_O$ . It follows that linear frequency scaling of  $L_1$  and therefore,  $L_2$  can be performed, i.e.:

$$L_1 = \frac{A_5}{F_O} \quad \text{and} \quad L_2 = \frac{A_6}{F_O}.$$

### Octave Bandwidth Design

Numerous 2-stage octave bandwidth CMRTPS's have been simulated using 4-finger Lange couplers—on a 25

mil thick alumina substrate, having a dielectric constant,  $\epsilon_r = 9.8$ .

The physical length of the Lange coupler dictates its frequency of maximum coupling,  $F_C$ . In order to obtain the optimum performance over the octave bandwidth,  $F_C$  is set slightly higher than the mid-band frequency. The reason for this increase will be discussed later.

Values for the previously defined empirical constants:  $A_1$ ;  $A_2$ ;  $A_5$ ;  $A_6$ ;  $k_1$  and  $k_2$  have been found which result in a phase error of less than  $\pm 2.4^\circ$  and a maximum amplitude error of less than  $\pm 0.21$  dB over the complete octave bandwidth—for relative phase shift levels up to and exceeding  $180^\circ$ .

These values are given in Table I. It should be noted that these values apply only to the type of directional coupler and substrate defined previously.

Quick design equations, based on the above tabulated values, are given below:

$$L_1 = \frac{3.9}{F_O}; \quad C_1(0) = \frac{48}{F_O}; \quad C_{C1} > \frac{960}{F_O}$$

$$L_2 = \frac{2.55}{F_O}; \quad C_2(0) = \frac{8.55}{F_O}; \quad C_{C2} = \frac{13.5}{F_O}$$

and

$$F_C = 1.087F_O$$

where  $L$  has units of [nH];  $C$  has units of [pF];  $F_O$  has units of [GHz].

#### Ku-Band Example

The following example of a 2-stage *Ku*-Band CMRTPS, with a mid-band frequency of 15 GHz, will serve to illustrate the above design techniques. The complete circuit diagram for a 2-stage CMRTPS, without a SIL is illustrated in Fig. 4. The component values for the reflection terminations and the finger dimensions for the 4-finger Lange couplers are given in Table II.

The following empirical expressions for  $\epsilon_{ee}(f)$ ,  $\epsilon_{eo}(f)$ ,  $C_{oe}(f)$  and  $C_{oo}(f)$  have been determined for the 4-finger Lange coupler, based on the results of Childs [15]:

$$\epsilon_{ee}(f) = 6.476 + 2.995 * 10^{-2}f + 8.469 * 10^{-4}f^2 - 1.739 * 10^{-5}f^3$$

$$\epsilon_{eo}(f) = 5.351 - 9.814 * 10^{-4}f - 1.976 * 10^{-4}f^2 + 4.317 * 10^{-6}f^3$$

$$C_{oe}(f) = 75.352 + 8.883 * 10^{-2}f + 1.2269 * 10^{-3}f^2 + 6.625 * 10^{-6}f^3$$

$$C_{oo}(f) = 399.28 - 6.323 * 10^{-2}f - 1.715 * 10^{-2} + 5.707 * 10^{-4}f^3 - 4.777 * 10^{-6}f^4$$

where  $C_{oe}$  and  $C_{oo}$  are in [pF] and  $f$  is in [GHz]. The above expressions are valid for frequencies up to 33 GHz.

TABLE I  
EMPIRICAL CONSTANTS FOR THE OCTAVE BANDWIDTH DESIGN

$A_1$	$A_2$	$A_5$	$A_6$	$k_1$	$k_2$
0.368	1.1348	3.900	2.550	>0.950	0.388

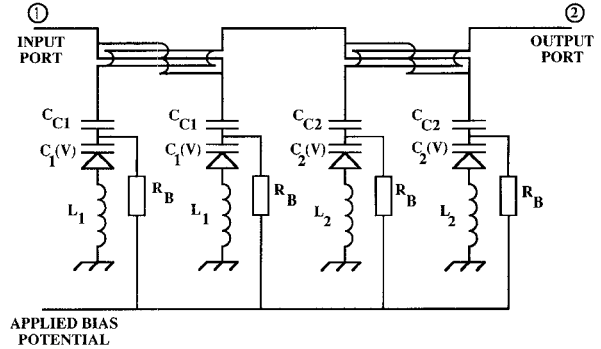


Fig. 4. Complete circuit of a 2-stage cascaded-match reflection-type phase shifter.

The resulting power and phase characteristics for the coupler are shown in Fig. 5. These characteristics are identical to those obtained using EESOF's "Touchstone" package.

The simulated frequency responses for this CMRTPS, using GaAs chip varactors (with  $\gamma = 1.0$ );  $R_{S1} = R_{S2} = 0.5 \Omega$  and  $Z_O = 50 \Omega$ , are shown in Fig. 6.

With reference to Figs. 3(c), 5 and 6(a), the negative gradients in Fig. 3(c) have been compensated for, within the octave bandwidth.

As frequency decreases below  $F_1$  or increases above  $F_2$ ,  $\Delta \angle S_{21}|_{PS}$  begins to increase from its level state. This is the result of the higher-order inter-stage reflections becoming more significant—as the return losses at port 2 of RTPS-1 and port 1 of RTPS-2 decrease—due to the increase in the 3 dB coupling imbalance.

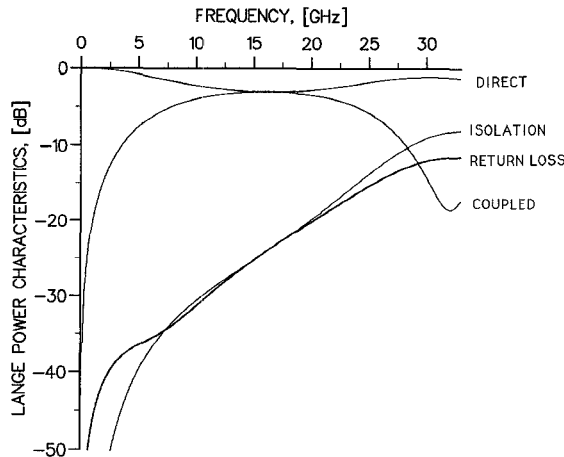
Eventually, a point is reached where  $(\partial \Delta \angle S_{21} / \partial \omega)|_{PS} \equiv 0$ , beyond which the decrease in the power coupling of the directional coupler dominates. With less reflected power emerging from port 2 of RTPS-1—and, therefore, RTPS-2—the  $\Delta \angle S_{21}|_{PS}$  response begins to rapidly decrease. The two resulting peaks must be located well away from octave band edges, as the changing gradients of their skirts are difficult to predict and, therefore, control.

With the types of directional couplers and substrates defined previously, the first peak is centered at approximately  $0.31 F_C$ ; while the second peak is at approximately  $1.67 F_C$ . The second peak is much greater in amplitude, compared to the first peak, due to the decreased isolation at that peak.

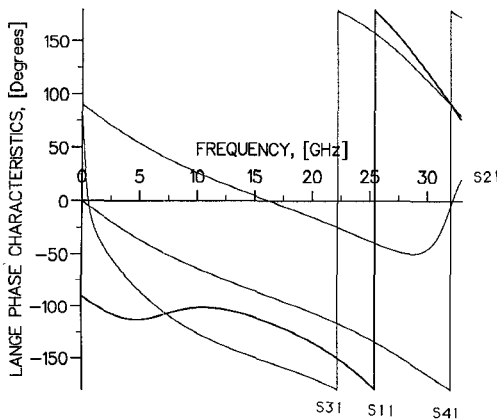
It has been found that the range of maximum  $\Delta \angle S_{21}|_{PS}$  flatness has a centre located just below  $F_C$ . This is because the decrease in isolation, as frequency increases, causes the 3 dB coupling imbalance to increase more at  $F_C + \Delta F_C$  than at  $F_C - \Delta F_C$ , where  $\Delta F_C$  is a small frequency offset from  $F_C$ . Therefore, in order to obtain the maximum

TABLE II  
COMPONENT VALUES AND FINGER DIMENSIONS FOR THE Ku-BAND EXAMPLE

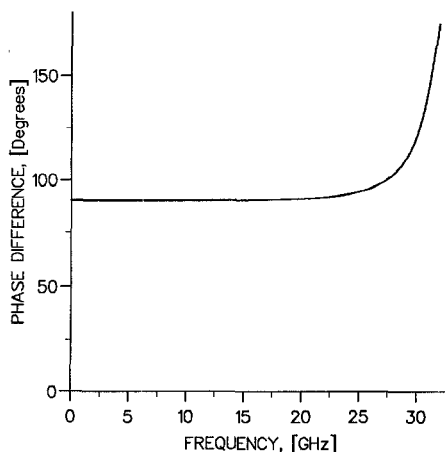
$L_1$ [nH]	$C_1(0)$ [pF]	$C_{C1}$ [pF]	$L_2$ [nH]	$C_2(0)$ [pF]	$C_{C2}$ [pF]	Length [ $\mu\text{m}$ ]	Width [ $\mu\text{m}$ ]	Separation [ $\mu\text{m}$ ]
0.26	3.2	>64	0.17	0.57	0.9	1850	65	53



(a)



(b)



(c)

Fig. 5. Four-finger Lange coupler characteristics. (a) Power. (b) Phase. (c) Phase difference between the coupled and direct port signals.

bandwidth, the mid-band frequency should be negatively offset slightly from  $F_C$ , i.e.,  $F_O \approx 0.85 F_C$  with the type of directional coupler and substrate defined previously.

For the octave bandwidth design presented here, an offset of  $F_C = 1.087 F_O$  was found to provide the minimum error in both the phase and amplitude and maximum return loss across the bandwidth.

A complete breakdown in the  $\Delta \angle S_{21}|_{P_S}$  response is found around  $1.72 F_C$ . This is due to the rapid increase in deviation from phase quadrature, as seen in Fig. 5(c).

#### IV. DISCUSSION

In order to realize the design of the 2-stage octave bandwidth CMRTPS, the tuning requirements of the CMRTPS and the varactor technologies available must be considered. Only varactors with a mesa type construction will be considered in detail, as accurate high-frequency models for planar MESFET type varactors are not available.

Varactor-type devices can be categorized by the semiconductor material from which they are made and by their doping profiles. Devices made from silicon have a barrier potential,  $\phi = 0.7 - 0.8$  V, while GaAs have  $\phi = 1.2 - 1.3$  V. In general, the Q-factor of a GaAs device is approximately four times that of an equivalent silicon device. As a result, the phase shifter would have a much better insertion loss performance if GaAs varactors were employed. Q-values in excess of 75, at 10 GHz, are commercially available for GaAs chip varactors with a  $C(4\text{ V}) = 0.5$  pF junction capacitance and a total series resistance,  $R_{SV}(4\text{ V}) = 0.42 \Omega$ .

The total series resistance of a mesa type varactor is the result of: an undepleted epitaxial region resistance, Ohmic contact resistance, spreading resistance between the epitaxial and substrate layers, and the substrate layer resistance. The undepleted epitaxial region resistance is a function of the applied bias potential. When the undepleted epitaxial region decreases, due to increases in the applied bias potential,  $R_{SV}(V)$  decreases. Since the total series resistance of the varactor makes-up the bulk of the total series resistance of the reflection termination,  $R_S(V)$  also decreases.

The simulations in the previous section have assumed a voltage-invariant  $R_S$ . However, with state-of-the-art high-Q GaAs varactors, this is a good approximation, for most applications.

A device with  $\gamma = 0.5$  is defined as having an abrupt junction (AJ), while those with  $\gamma > 0.5$  are defined as having a hyper-abrupt junction (HAJ). The value of  $\gamma$  depends on the doping profile of the active layer. Traditionally, HAJ devices have voltage dependant  $\gamma$  values. How-

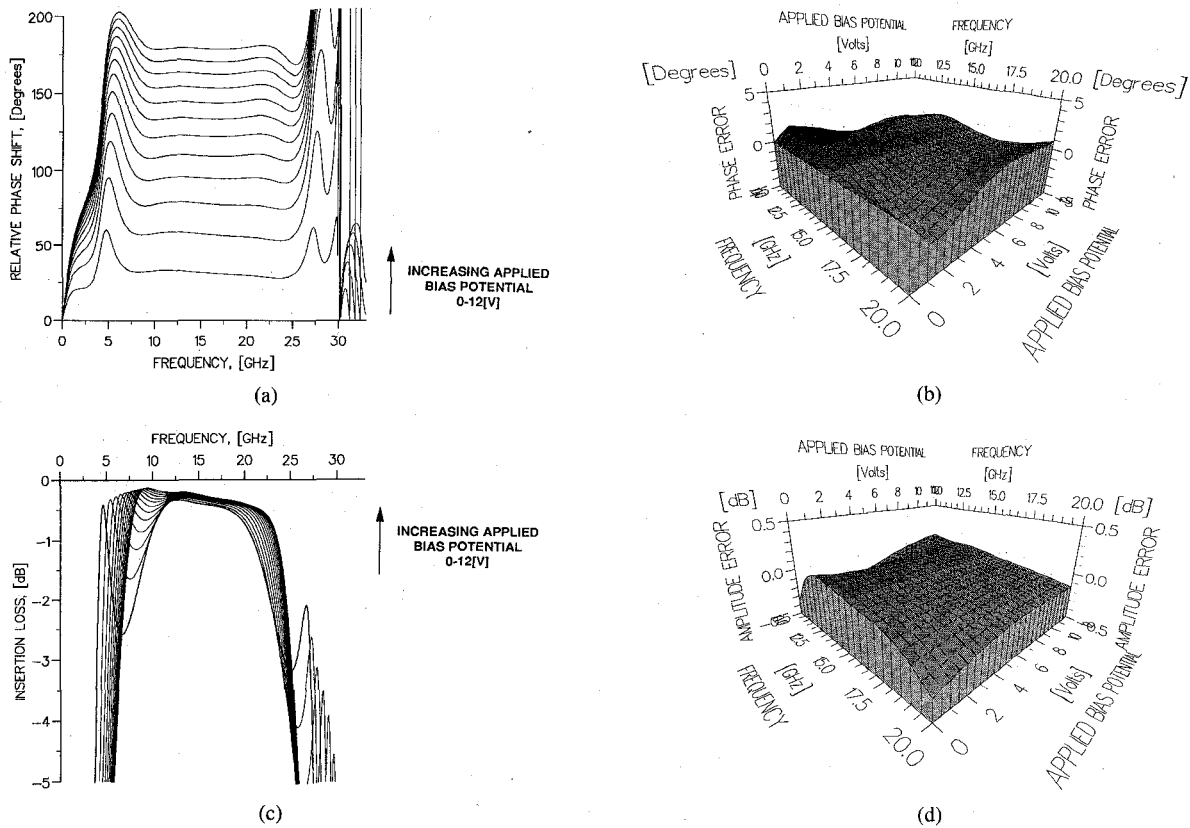


Fig. 6. Simulated performance of a 2-stage *Ku*-band CMRTPS. (a) Relative phase shift. (b) Phase error. (c) Insertion loss. (d) Amplitude error.

ever, constant- $\gamma$  varactors are now commercially available. In general, the Q-factor of a HAJ varactors is approximately a factor of 2 to 3 higher than equivalent AJ varactors.

The tuning curve of the 2-stage octave bandwidth CMRTPS is given in Fig. 7. From Fig. 7, the  $\Delta \angle S_{21}|_{PS} - V$  tuning characteristics for various types of varactor are shown in Fig. 8. It can be deduced from Fig. 7 and seen in Fig. 8 that near linear tuning can be achieved, without any form of bias equalization network, with HAJ varactors having  $\gamma = 2.0$ .

Since the breakdown voltage,  $V_B$ , of most varactor devices is less than 40[V], it will be apparent, from Fig. 8, that only constant- $\gamma$  GaAs HAJ varactors will be suitable for achieving the 180° level of relative phase shift. If AJ varactors are to be employed, a wideband 90° binary phase shifter could be added to achieve the 180° level, resulting in a slight performance degradation.

Assuming  $(V_B - V_2) \geq (\phi + V_1)$ , if the RF power level exceeds a critical saturation value,  $P_{MAX}(V)$ , forward bias rectification occurs and unwanted harmonics will be generated. For a 3 dB directional coupler and a 3 dB coupling imbalance of  $\pm 1.5$  dB, the value of  $P_{MAX}(V)$  is given as

$$P_{MAX}(V) = \frac{0.354}{Z_0} \cdot (\phi + V)^2$$

$$\text{for } -\phi \leq V \leq (V_B - \phi)/2.$$

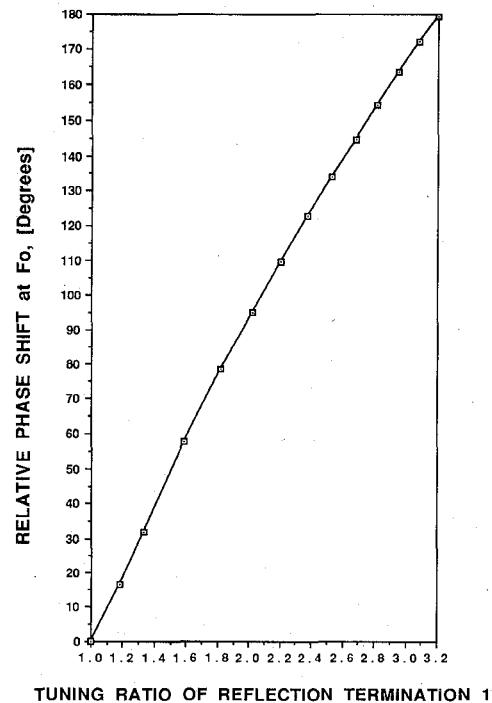


Fig. 7. Tuning curve for the 2-stage octave bandwidth CMRTPS.

Therefore,  $P_{MAX}(V)$  decreases as  $V$  decreases. For a 50  $\Omega$  system with  $V_1$  set to zero:  $P_{MAX}(V_1) = 10.7$  dBm for GaAs varactors and  $P_{MAX}(V_1) = 5.4$  dBm for Si varac-



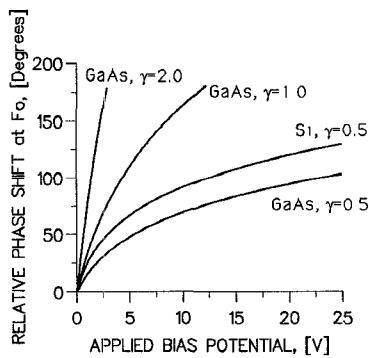


Fig. 8.  $\Delta \angle S_{21}|_{PS} - V$  tuning characteristics for various types of varactor.

tors. If the RF power level is to exceed  $P_{MAX}(V_1)$ , with a hybrid realization, the value of  $V_1$  must be increased to

$$V_1 = \left( \frac{Z_0 P_{MAX}(V_1)}{0.354} \right)^{1/2} - \phi$$

An increase in  $V_1$ , from zero, will force a subsequent increase in  $V_2$ , in order to achieve the 180° level—resulting in an increased performance degradation at the higher levels.

For a hybrid realization of the CMRTPS, Mesa-type varactors are generally used. Here, the parasitics associated with the chip varactors have a significant effect on the performance of the CMRTPS above 10 GHz. With the *Ku*-band design example, a factor of 2 degradation in the ideal phase error was found.

At present, a MMIC realization of the CMRTPS would generally use planar-type varactors. Here, varactors can be realized by connecting together the drain and source terminations of a standard MESFET—resulting in a single Schottky junction. The bias potential is then applied across the drain/source and gate terminations. An MMIC RTPS using this technique has been reported by Bianchi *et al.* [12].

MMIC varactors, resulting in large tuning ratios, can be produced using selective ion implantation (SII) techniques to nonuniformly dope the active region. At present, SII is comparatively a very expensive operation. However, a number of RTPS's have been reported using these devices [6], [8], [9].

The small parasitic reactances associated with MMIC varactors should result in a negligible degradation in the ideal phase error of the *Ku*-band design example.

## V. CONCLUSION

The novel technique of cascading matched RTPS's has been presented. With the three conditions for the reflection terminations satisfied, the bandwidth limitations of the nonideal 4-finger Lange coupler can be overcome to

realize high performance octave bandwidth 180° analog phase shifters.

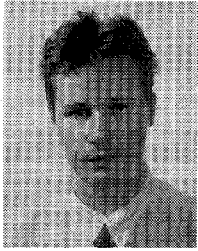
With high-Q GaAs HAJ varactors, the CMRTPS can achieve the 180° relative phase shift level with a low applied bias potential and exhibit excellent insertion loss and noise figure characteristics.

Due to the inherently small, yet well characterized, parasitics associated with a MMIC realization and with a good design layout, the measured performance should be close to that predicted.

The linear transmission phase and error characteristics make the 2-stage CMRTPS ideally suited for low power wide-band oscillators and beam forming network applications.

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